# GDP-B: Accounting for the Value of New and Free Goods

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*The welfare contributions of new goods and free goods are not well-measured in our current national accounts. We derive explicit terms for the contributions of these goods and introduce a new framework and metric, GDP-B which quantifies their benefits. We apply this framework to several empirical examples including Facebook and smartphone cameras and estimate their valuations through incentive-compatible choice experiments. We find that including the gains from Facebook adds 0.05 to 0.11 percentage points to welfare growth per year while improvements in smartphones adds approximately 0.63 percentage points per year. (*JEL *C43, D60, E23, O3, O4)*

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*"The welfare of a nation can scarcely be inferred from a measure of [GDP]."*

*– Simon Kuznets, 1934*

## I. Introduction

We develop a new framework for measuring welfare change in the presence of new and free goods.[1] The increased proliferation of such goods is a key characteristic of the digital economy. New goods appear with increasing rapidity,[2] and digital goods (such as information and entertainment services) are increasingly available at zero price, reflecting their low marginal costs of replication and distribution. Even when free goods have an implicit price, this price is not usually observed, so a price of zero is used in the national accounts.[3] Thus, the positive quantities of these goods that are consumed have a measured price of zero and measured value of zero in the conventional national accounts even if they create considerable consumption value for consumers. A related difficulty arises in assessing the welfare contributions of new goods since there is no observed price for the period before their appearance. Despite the increasing relevance of new and free goods, the value to consumers is not reflected in standard statistical agency reports for GDP or derivative metrics like productivity, which are typically defined in terms of GDP.

Welfare measurement is among the most fundamental questions for economics. Despite the appeal of using GDP as a proxy for welfare (Dynan and Sheiner, 2018), and its widespread use for this purpose among policymakers, journalists and economists, it does not correctly reflect the benefits from the introduction of new and free goods. GDP, as conventionally measured, may entirely miss the benefits from the increased production and use of free goods, and it can even have the wrong sign when a free good replaces a good with a positive price, or vice versa. As digital goods proliferate, we risk increasingly misunderstanding the economy unless we update our metrics.

Our framework provides a means by which to understand the welfare contributions from new and digital goods. Our focus is on real consumption by households and the associated welfare gains, rather than production, which is properly the focus for conventional GDP metrics. We use our framework to derive an explicit term that is the marginal value of a new good on welfare change. This can shed light on the debate regarding the potential of

[1] Throughout this paper, we use the word "goods" to refer to goods and services collectively.
[2] Goolsbee and Klenow (2018, Table 3), using Adobe Analytics data on online transactions for millions of products across many different categories, find that roughly half the sales volume online for 2014-2017 is for products that did not exist in the previous year.
[3] See Nakamura, Samuels and Soloveichik (2016) and Brynjolfsson and Oh (2012) for examples of how to think about the valuation of "free" media.

the digital economy to generate productivity, economic growth and welfare gains, and the paradox implicit in the gap between the rise of the digital economy and the fall of productivity as conventionally measured.[4] In particular, if measurement is lacking, through methodological challenges, statistical agency budgets or data availability, then we are severely hampered in our ability to understand the impact of new technologies, goods on the economy, and consequently the prospects for future productivity, economic growth and welfare improvements.[5] The pandemic-driven increase in remote work and the associated digital technologies places new emphasis on the importance of measuring their impact on welfare and the economy.[6]

Our focus on real consumption creates a scalable and robust companion to expenditure-based GDP.[7] A problem in assessing the full impact of the introduction of a new good on real GDP growth using only conventional metrics is that it would require that national statistical offices recalculate their estimates of real GDP including the consumption of new goods with estimated Hicksian reservation prices for the period before they are sold in positive quantities[8] However, we are able to use our framework to derive a close approximation to the addition to real GDP growth that would be required to account for the welfare gains from the introduction of a new good, without having to recalculate the official GDP numbers published by national statistical offices.[9]

Free goods are addressed through generalizing the standard microeconomic model of household cost minimization. It is then possible to derive welfare change and real GDP growth adjustment terms to allow for there to be free goods. Our new metric is labelled

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[4] Among others, see, for example, Gordon (2016) and Cowen (2011) giving a pessimistic view and Sichel (2016), Mokyr, Vickers and Ziebarth (2015) and Brynjolfsson and McAfee (2011, 2014) giving a more optimistic view. Brynjolfsson, Rock and Syverson (2019) review several explanations for this “modern productivity paradox”.

[5] Among others, see, for example, Feldstein (2017), Groshen *et al*. (2017), Hulten and Nakamura (2017), Syverson (2017), Ahmad and Schreyer (2016), Byrne, Fernald and Reinsdorf (2016), Brynjolfsson and Saunders (2009), Brynjolfsson and Oh (2012), Brynjolfsson, Rock and Syverson (2017), Greenstein and McDevitt (2011), Brynjolfsson, Eggers and Gannamaneni (2018) and Brynjolfsson, Collis and Eggers (2019).

[6] According to Brynjolfsson et al (2020), the number of Americans working from home more than tripled from 14.6% to 48.7% in the first half of 2020.

[7] US GDP is measured using the expenditure approach, *not* the production approach; see BEA (2015; 4). Hence, while e.g. salaries paid to workers producing digital products will be collected by the BEA, they are not directly used in constructing GDP, except for the salaries of developers of software databases classified as contributing to own-account investment. Similarly, for the production costs and revenues from advertising, which support the production of many digital products, which are only reflected indirectly through the final prices paid by consumers..

[8] The reservation price of a good is the price which would induce a utility maximizing potential purchaser of the product to demand zero units of it. See Hicks (1940), Diewert (1980), Hausman (1981, 1996), Feenstra (1994), Diewert, Fox and Schreyer (2018), and Diewert and Feenstra (2022).

*GDP-B,* as it captures the *benefits* associated with new and free goods and thus goes beyond GDP.[10] In addition, our calculations of GDP-B provide a straightforward way to derive a corresponding productivity metric, *Productivity-B* which simply uses GDP-B as its numerator.

Our focus is on developing a methodology to capture things which are not currently measured in GDP. The aim is to show how the national accounts can be expanded so that national statistical offices can either produce satellite accounts for GDP-B, possibly as part of a spectrum of GDP variants as being considered by the UK Office for National Statistic (Heys et al. 2019), or so that our GDP-B methodology can be adopted in the construction of official GDP.[11] The methodology may also prove useful for measuring existing components of GDP, where standard methodologies already exist. We leave this to future research and focus on the problem of measuring things which are currently not measured.

Many new goods have appeared in the past and their contributions to welfare from their introduction to consumer consumption baskets have not been measured. We cannot address past omissions, as we do not have estimates of consumers' valuations of, e.g., the introduction of radio or television programming. Our goal is to provide a framework so that such omissions do not need to happen in the future.

We demonstrate the application of our framework and quantify these welfare and GDP growth adjustment terms using several empirical examples of free digital goods. Specifically, we draw on the work of Brynjolfsson, Collis and Eggers (2019) who developed an approach to directly estimate consumer welfare by running massive online choice experiments. They explored both hypothetical and incentive-compatible choice experiments to estimate willingness to accept values for giving up access to a good. While hypothetical choice experiments can suffer from bias, incentive aligned choice experiments,

[10] For more on the "beyond GDP" literature, see e.g. Stiglitz, Sen and Fitoussi (2009), Jones and Klenow (2016), Coyle and Mitra-Kahn (2017), Corrado *et al.* (2017), OECD (2018) and Jorgenson (2018).

[11] Heys et al. (2019) present options being considered by the UK Office for National Statistics, which include incorporating welfare adjustments for private and publicly provided free goods. Our approach in this paper provides a way of doing this, which they acknowledge: "This would deliver an estimation of the value consumers feel they receive". Paul Schreyer (OECD Director of Statistics) notes the following (Schreyer 2022): "In an inspiring paper, Brynjolfsson, Collis, Diewert, Eggers and Fox…accomplish two important tasks in regard to the measurement of the digital economy. First, they derive explicit index number expressions for the contributions of free products to welfare change. Second, the authors quantify these contributions in the case of several free digital services…." (p. 2) "A useful way forward at this junction is the systematic and periodic development of measures of household production and consumption outside the current SNA boundaries but inside a framework of satellite accounts so that accounting concepts are adhered to, results can be compared with established national accounts aggregates and experimental aggregates…can be constructed." (p. 23)

which make participants' choices consequential, have been shown to be externally valid (Ding, Grewal and Liechty 2005; Carson, Groves and List 2014; Bishop et. al. 2017).

We therefore constructed incentive-compatible discrete choice experiments to estimate the potential impact on welfare growth by Facebook, a free social networking service which had rapid diffusion and quickly accumulated many diverse users. We ran our experiments on a representative sample of the US internet population recruited through an online survey panel. We use the results to provide estimates of the adjustments to welfare change and real GDP-B growth from Facebook's launch in 2004 through 2017.

In addition, in a laboratory setting in the Netherlands, we also ran incentive-compatible choice experiments to estimate the consumer welfare created by several other popular digital goods, including Instagram, Snapchat, Skype, WhatsApp, digital Maps, LinkedIn, Twitter as well as Facebook.[12] Although we did not have a representative sample of the population in the laboratory, our results are indicative of the approximate size of the adjustment term to real GDP-B growth which would need to be added to account for the welfare gain from these digital goods.

We also derive and apply a method for adjusting for quality changes from new goods or features in calculating GDP-B growth so that welfare changes are properly inferred. This issue is particularly acute for smartphones which have added new features that substitute (to varying degrees) a panoply of other devices including cameras, GPS, landline phones, gaming consoles, e-book readers, personal computers, video and audio players, maps/atlases, alarm clocks, calculators and sound recorders,[13] as well as numerous new capabilities that previously were unavailable at any price such as real-time traffic and various types of social networking and messaging applications.

What is more, new features are added frequently to smartphones and quality of existing features changes rapidly. In fact, application developers conduct thousands of A/B tests every day and tweak features to improve user experience. Groshen *et al*. (2017) discuss

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[12] There can be complications with bundling of services for some products, resulting in a difficulty in separating valuations. It is also possible that the free goods are provided with paid goods, which have markups that cover the cost of providing the free goods. Neither case seems applicable to the digital products examined in this study.

[13] See https://www.huffingtonpost.com/steve-cichon/radio-shack-ad_b_4612973.html (accessed Feb 10, 2019) and also Hal Varian's presentation at Brookings (https://www.brookings.edu/wp-content/uploads/2016/08/varian.pdf, accessed March 19, 2019).

how the US Bureau of Labor Statistics (BLS) seeks to adjust for quality changes using hedonic methods. However, they explain that the hedonic approach was historically ruled out for smartphones since the set of relevant characteristics for the hedonic models constantly keep on changing. Subsequently, the US BLS commenced a set of hedonic quality adjustments for smartphones from January 2018,[14] though such explicit hedonic quality adjustment is still very limited internationally, with the UK ONS being a standout early adopter of this approach for smartphones, commencing in 2011 (see Wells and Restieaux (2014)).

Therefore, we conduct an incentive compatible BDM lottery study (Becker, DeGroot, and Marschak 1964) in a university laboratory in the Netherlands to elicit consumers' valuations for smartphone cameras. We find that there is a large difference between the contribution of smartphone cameras towards conventional GDP and the welfare generated by these cameras for consumers as reflected in GDP-B. As a result, not accounting for quality adjustments in smartphones leads to a significant underestimate of welfare growth.

Several objections to our approach may be raised, as follows:

(i) *Free digital goods are funded by advertising, which is measured in GDP.* First, many important free goods, such as Wikipedia, open source software and most blogs and online videos are not. Second, as noted by Spence and Owen (1977), expenditures on advertising do not reflect the benefits to consumers the way conventional prices do.[15] Thus, simply measuring expenditures on advertising will not provide an accurate estimate of the welfare gains to consumers from the goods.

(ii) *Free digital goods are not really "free" as consumers effectively barter for these goods with their data or with their attention to bundled advertising.* That may be true, but such bartered goods are not captured in conventionally-measured GDP either. In contrast, our approach will reveal the net benefits of such goods, after

[14] See "Measuring Price Change in the CPI: Telephone hardware, calculators, and other consumer information items", available at https://www.bls.gov/cpi/factsheets/telephone-hardware.htm.

[15] The intuition is that advertisers pay for "eyeballs" -- number of views they purchase – but are indifferent about how much benefit, if any, each individual viewer gets from the accompanying content they consume. Very valuable content, with relatively few viewers, will generate less advertising revenue than widely-viewed but low-value content.

subtracting any costs that consumers assign to the time or data that they give up in exchange.

(iii) *It is hard to think about price and quantity concepts for free goods.* We can choose what unit we wish to define for each free good and then the price as the marginal willingness to forego one unit of the good. For instance, in the case of the Facebook in this paper, we define the quantity for each user as one month of access and use as our price the willingness-to-pay for one month of access. For other goods, like say, Wikipedia, one might ask about access for certain period of time, for access to all or to a certain subset of posts, to certain modes of access or to other factors which affect consumption and well-being.

(iv) *GDP is a measure of production, not welfare or consumption*. This is exactly why additional metrics needed. While, in some cases, under stringent assumptions, one can infer changes in welfare from changes in GDP, this is not true in general, particularly when free or new goods are involved. Thus, while GDP provides a wealth of essential insights about the economy, it is not a sufficient statistic for all economic questions.

(v) *The national accounts seek to match all changes consumption with changes in production.* The expenditure approach to measuring GDP, as used in constructing official U.S. GDP (BEA (2015; 4), includes household final consumption. If households increasingly substitute from consumption goods with market prices to goods without market prices, real consumption is not falling, it is becoming unmeasured. The question arises as to where the corresponding production that corresponds to the consumer valuations is coming from. Inspired by prior draft of this paper, Paul Schreyer (OECD Director of Statistics) proposes that it is from households using the free digital goods to produce own-account leisure services, using time, labor and capital; Schreyer (2022). Hence, there is unmeasured production that corresponds with the unmeasured consumption that is the focus of our paper.

(vi) *If consumers pay zero for free goods, why isn't the marginal value of free goods equal to zero?* The marginal value can be zero, but the monetary compensation that is needed by the household to maintain utility without any of the free good is positive. Although it has a market price of zero, the value of a marginal unit of a free good to the consumer is equal to the cost reduction it provides in achieving a certain utility level. If consumers can reach their satiation point for the consumption of the good, then the marginal valuation will go to zero in our framework. This is illustrated in Figure 1b of Section 2. That said, in a model with a household time constraint, the marginal value of a free good, including the cost of time, will be positive.[16] However, what we need in our framework is simply that there be a positive monetary compensation needed by households to maintain utility without the free good.

(vii) *Why the focus on the consumption of digital goods and not on other uses of our time?* It would be useful and natural to extend our approach to these questions in future work. Currently, other uses of our time in own-account production can include activities such as cooking dinners and mowing lawns. For measures of "GDP and beyond" (Eurostat 2010), ultimately the inclusion of a wide range of household production should be included in the national accounts. The U.S. already publishes a satellite account on household production that measures unpaid work done in the home; see Kanal and Kornegay (2019). Hence, significant progress has been made in empirically measuring traditional home production. This is not the case so far for home production of consumption services using digital goods. That said, in aggregate across the population, traditional household production activities are likely to have changed much less over time than the home production from using digital goods, such as Facebook, digital maps, Instagram, and WhatsApp.

(viii) *What about the consumer losses from the disappearance of goods which have been replaced by digital goods?* When, for example, WhatsApp largely replaces conventional telephone texting, then the traditional GDP already captures the fall

[16] See Neary and Roberts (1980) and their related concept of "virtual prices" for rationed goods.

in disappearing value of these telephone services but misses the gains in consumer value from WhatsApp. Hence, measures of economic growth based on GDP are biased downward for this case, and related cases.

The rest of the paper is organised as follows. The next section sets out some preliminary definitions that will be used in the subsequent sections. Section 2 introduces our first version of GDP-B, using what we call the total income approach. This can be shown to approximate a true measure of GDP growth calculated conventionally, but with reservation prices for the new free digital goods. This approach does not rely on consumer surplus arguments, and can be implemented without requiring a national statistical office to recalculate their index of real GDP. Section 4 extends standard welfare concepts to the case of free goods and introduces a broader extension of GDP. Section 5 provides the empirical examples of Facebook and other popular free digital goods to quantify adjustments to welfare change and GDP-B growth for not accounting for these goods. Section 6 presents results from the smartphone study to highlight potential biases due to not performing quality adjustments created by new or improved features. Section 7 concludes with a summary and some implications. Proofs are in the Appendix and supplementary results are provided in an online appendix.

## II. Preliminaries

Consider a household whose preferences over $N$ market goods and $M$ goods that are available to the household with no visible price can be represented by the utility function $f(q,z)$ where $q \geq 0_N$ and $z \geq 0_M$ are vectors which represent the consumption of market goods and of free goods respectively. We assume that $f(q,z)$ is defined over the nonnegative orthant in $R^{N+M}$ and has the following properties: (i) continuity, (ii) quasiconcave in $q$ and z and (iii) $f(q,z)$ is increasing in all components of $q$ and $z$.

We define two cost or expenditure functions that are dual to $f$. The first cost function is the consumer's *regular cost function*, $C(u,p,w)$, that is the solution to the following cost minimization problem which assumes (hypothetically) that the household faces positive prices for market and free goods so that $p \gg 0_N$ and $w \gg 0_M$:[17]

$$(1) \qquad C(u,p,w) \equiv \min_{q,z}\{p \cdot q + w \cdot z : f(q,z) \geq u, q \geq 0_n, z \geq 0_M\}.$$

We also define the household's *conditional cost function*, $c(u,p,z)$ where the household minimizes the cost of market goods needed to achieve utility level $u$, conditional on having the vector $z \geq 0_M$ of free goods at its disposal:

$$(2) \qquad c(u,p,z) \equiv \min_{q}\{p \cdot q : f(q,z) \geq u, q \geq 0_N\}.$$

It can be shown (using feasibility arguments) that $c(u,p,z)$ has the following properties where $u \in$ Range $f$, $p \gg 0_N$, and $z \geq 0_M$: (i) for fixed $u$ and $z$, $c(u,p,z)$ is nonnegative and linearly homogeneous, concave and nondecreasing in $p$ and (ii) for fixed $u$ and $p$, $c(u,p,z)$ is nonincreasing and quasiconvex in $z$. If in addition, $f(q,z)$ is linearly homogeneous in $q$ and $z$ (the homothetic preferences case), then $c(u,p,z)$ is linearly homogeneous in $u, z$ for fixed $p$.

The regular cost function minimization problem defined by (1) can be decomposed into a two-stage minimization problem using the conditional cost function in (2), as follows:

$$(3) \qquad C(u,p,w) = \min_{z}\{c(u,p,z) + w \cdot z : z \geq 0_M\}$$

Suppose $z^* \geq 0_M$ solves this cost minimization problem and suppose further that $c(u,p,z^*)$ is differentiable with respect to the components of $z$ at $z = z^*$. Then the first

[17] We assume u is in the range of f(q, z).

order necessary conditions for $z^*$ to solve the cost minimization problem imply that the following first order conditions hold:

$$\nabla_z c(u, p, z^*) = -w. \tag{4}$$

With $z = z^*$, we can use the conditional cost minimization problem defined by (2) to find a $q$ solution; i.e., $q^*$ is a solution to $\min_q \{p \cdot q : f(q, z^*) \geq w, q \geq u\}$. It can be seen that $(q^*, z^*)$ is a solution to the regular cost minimization problem defined by (1) so that $C(u, p, w) = p \cdot q^* + w \cdot z^*$.

Thus the imputed marginal valuations $w \equiv -\nabla_z c(u, p, z^*) \geq 0_M$ are appropriate prices to use when valuing the services of free goods in order to construct cost of living indexes or measures of money metric utility change. That is, although it has a market price of zero, the value of a marginal unit of a free good to the consumer, $w_m$, is equal to the cost reduction it provides in achieving utility level $u$.

Due to the fact that $c(u, p, z)$ is decreasing and quasiconvex in the components of $z$, the marginal price for an additional unit of $z_m$, $w_m(u, p, z) \equiv \partial c(u, p, z)/\partial z_m$, will be nonincreasing in $z_m$; i.e., it will usually decrease as we add extra units of $z_m$ to the household's holdings of free goods.[18]

If the household holds the amount $z^* > 0_M$ of free goods, then we can develop a willingness-to-accept measure as follows. Let $q^*$denote the household's observed market goods consumption vector and we again assume that they face the vector of market goods prices $p$. Let $u^* \equiv f(q^*, z^*)$. We assume that the market cost of achieving its utility level $u^*$ is minimized so that $p \cdot q^* = c(u^*, p, z^*)$. Now suppose that the household disposes of its vector of free goods $z^*$. The amount of income that the household would require to attain the same level of utility $u^*$ is increased to $c(u^*, p, 0_M) > c(u^*, p, z^*)$. Then the consumer should be willing to sell its free goods for the amount $c(u^*, p, 0_M) - c(u^*, p, z^*)$,

[18] If consumers can have the free good in unlimited amounts, then its price will be zero. However, even if the price is zero, if quality improves, the marginal willingness to pay for the improved quality will be positive, hence $w_m(u, p, z)$ will be greater than zero. We thank Marshall Reinsdorf for this point.

i.e. the amount that they would accept for giving up the free goods. Thus, we define the *"global" willingness-to-accept function*, for the disposal of $z^*$ as follows: [19]

$$W_A(u^*, p, z^*) \equiv c(u^*, p, 0_M) - c(u^*, p, z^*) \qquad (5)$$
$$= c(u^*, p, 0_M) - p \cdot q^* = m,$$

where $m$ is the amount of monetary compensation that is required to keep the household at the utility level $u^*$ without using any of the free good. For small changes in components of $z^*$, it can be seen that (5) is a discrete approximation to the marginal valuation price vector $w \equiv -\nabla_z c(u, p, z^*)$ that was defined by (4).

What is important for our paper is that $m$ in equation (5) be positive, not that $w$ be positive. If we only assume that $f(q,z)$ is quasiconcave in $(q,z)$, (and $f$ is strictly increasing in its arguments and is continuous), then we can establish that $c(u,p,z)$ is strictly decreasing and quasiconvex in $z$. Thus if the derivatives of $c(u,p,z)$ with respect to the components of $z$ exist, they will be negative or possibly equal to 0 at isolated points. But $c(u,p,z)$ will definitely be strictly decreasing in the components of $z$ which will imply that $m$ is positive, which is what is required, not that the marginal value is positive.

We illustrate this in figures 1a and 1b, where we have only two commodities, $q_1$ and $z_1$, with the indifference curve for utility level $u^*$ indexed by $u \equiv f(q_1, z_1)$.[20] In Figure 1a we consider that $z_1$ is not a free good but has a positive market price of $w_1 > 0$. Figure 1b shows the case where $z_1$ is a free good, so that $w_1 = 0$. In both figures, the optimal consumption bundle $(q_1^*, z_1^*)$ is represented by point $A$.

[ Insert Figure 1a Here ]

In Figure 1a, the price for a unit of $q_1$ is $p_1$ and the price for a unit of $z_1$ is $w_1$, so that the slope of the budget line is $-w_1/p_1$. This is tangent to the indifference curve at point $A$, where $u^* = f(q_1^*, z_1^*)$. At $A$ the consumption bundle is $(q_1^*, z_1^*)$, the solution to the

[19] See Brynjolfsson, Collis, Diewert, Eggers and Fox (2019) for a definition of the analogous willingness-to-pay measure.
[20] This figure is adapted from Diewert, Fox and Schreyer (2022).

conditional cost function $c(u^*, p_1, z_1^*)$ The reservation price that drives demand for $z_1$ to zero is $w_1^R$, so that the slope of the indifference curve at the point $(q_1^*, 0)$ is $-w_1^R/p_1$, and $q_1^{**}$ is the solution to the conditional cost function $c(u^*, p_1, 0)$. Hence we have $f(q_1^*, z_1^*) = f(q_1^{**}, 0) = u^*$, i.e. both points $A$ and $B$ are on the same indifference curve.

We can draw a line between points $A$ and $B$ and this will have the slope $-W_A^C/p_1$, where $W_A^C \equiv m/z_1^*$, the average income per unit of $z_1$ that the consumer needs in compensation in order to have the same level of utility, $u^*$, if they have to give up consuming $z$. This follows from equation (5).

[ Insert Figure 1b Here ]

In Figure 1b, $z_1$ has a market price of zero so that $w_1 = 0$. Here the consumption point $A$ is the satiation point for consumption of $z_1$ at a zero price, such that the marginal value of an additional unit of $z_1$ is zero, i.e. $\partial c(u, p_1, z_1)/\partial z_1 = -w_1 = 0$. The line connecting points $A$ and $B$ again has a slope that can be expressed as $-W_A^C/p_1$. Hence, although the marginal value at point $A$ is zero, to move from point $A$ to point $B$, where no $z_1$ is consumed, the consumer would need to receive compensation equal to $W_A^C z_1 = (m/z_1^*)z_1^* = m$.

That is, in terms of equation (5), with our two good example we have the following:

$$
\begin{aligned}
W_A(u^*, p_1, z_1^*) &\equiv c\left(u^*, p_{1,} 0\right) - c(u^*, p_1, z_1^*) \\
&= p_1 q_1^{**} - p_1 q_1^* \\
&= p_1(q_1^{**} - q_1^*) = m > 0,
\end{aligned}
\tag{6}
$$

as $q_1^{**} > q_1^*$, as can be seen from Figure 1b; to maintain the same level of utility, more of $q_1$ is needed if the free good $z_1$ is taken away. Thus $m > 0$, even though the marginal valuation of $z$ at $z_1^*$ is zero.

## III. Total Income and GDP Growth

Our first approach to adjusting GDP for free goods is as follows. From (**5**), we have the amount of additional income, $m$, that is needed when households are deprived of the goods so as to maintain utility at the same level as if the free goods are consumed. Hence, through reorganising (5) we can define *total income* (T) as follows:

$$T \equiv c(u^*, p, 0_M) = p \cdot q^* + m, \tag{7}$$

Equation (7) gives the total income required so that the level of utility, $u^*$, that can be attained with the consumption of market *and* free goods can be attained through the consumption of *only* market goods. That is, $p \cdot q^*$ is the observed income when the free goods are available and $m$ is the amount of extra income needed to maintain the level of utility through purchasing additional market goods if the free goods are unavailable.

We now take the value $p \cdot q^*$ in (7) to be nominal GDP, and then $m$ is the amount that needs to be added to capture the income equivalent that households receive from having access to free goods. Deflating the resulting nominal total income growth between periods 0 and 1, $T^1/T^0$, by the GDP deflator, $P$, gives real total income growth, GDP-$B_T$:

$$\text{GDP-B}_\text{T} \equiv (T^1/T^0)/P \tag{8}$$

A *maximum overlap* quantity index uses only data on (market) goods existing in both periods are used. This is what is usually used by national statistical offices in constructing GDP. The U.S. Bureau of Economic Analysis uses the Fisher quantity index, which can be defined as follows: $Q^F \equiv [(p^0 \cdot q^1/p^0 \cdot q^0)(p^1 \cdot q^1/p^1 \cdot q^0)\,]^{0.5}$, where $p^t$ and $q^t$ are price and quantity vectors, respectively, for periods $t = 0,1$.[21] Following Hicks (1940, p. 114), the correct, or "true" quantity index would capture the effects of new and disappearing goods through using reservation prices.

[21] The Fisher index has good properties from the axiomatic and economic approaches to index numbers, including being a superlative index number; see Diewert (1976).

PROPOSITION 1: GDP-$B_T$ in (8) can approximate a true Fisher quantity index of GDP calculated using reservation prices for new free goods.

The proof of the proposition is given in the Appendix.

Note that the calculation of GDP-$B_T$ does not require an estimate of the period 0 reservation price for any new good. Instead, from equations (7) and (8), all that is needed in addition to official published data on nominal GDP and the GDP deflator is an estimate of the additional nominal income needed in period 1, $m^1$, that would compensate for the loss of the good.

In this section we did not rely on consumer surplus arguments to derive our estimate of adjusted GDP using the total income approach, GDP-$B_T$. Instead, using a Fisher index example, we observed that GDP-$B_T$ is an approximation to the GDP that would be calculated by if a national statistical office included Hicksian reservation prices for the new free goods in their index. In the next section we look at an alternative adjustment method that uses standard measures of welfare change as the starting point.

## IV. Welfare Change and GDP Growth

For notational simplicity, we first consider having only regular market goods. Valid measures of utility change between two periods are the following Hicksian equivalent and compensating variations, respectively (Hicks, 1942):[22]

$$(9) \qquad Q_E(q^0, q^1, p^0) \equiv C(f(q^1), p^0) - C(f(q^0), p^0));$$

[22] These measures of overall quantity change are difference counterparts to the family of Allen (1949) quantity indexes in normal ratio index number theory. The Allen quantity index for reference price vector $p$ is defined as the ratio $C(f(q^1), p)/C(f(q^0), p)$. These are Hick's original definitions of equivalent and compensating variations. They are special cases of Samuelson's (1974) family of quantity variations, $Q_S(q^0, q^1, p) \equiv C(f(q^1), p) - C(f(q^0), p)$, for $p \gg 0_N$. Hence there is an entire family of cardinal measures of utility change, with one measure for each reference price vector $p$. Hicks (1946, 331-332) appears to provide an alternative definition of the equivalent variation as$C(f(q^1), p^1) - C\big(f(q^1), p^0\big)$ and the compensating variation as $C(f(q^0), p^1) - C(f(q^0), p^0)$. The existence of alternative definitions has caused significant confusion in the literature; see Diewert (1992, p. 567, footnote 10).

(10) $$Q_C(q^0, q^1, p^1) \equiv C(f(q^1), p^1) - C(f(q^0), p^1).$$

The variations defined by (9) and (10) are not observable (since $C(f(q^1), p^0)$ and $C(f(q^0), p^1)$ are not observable) but the following Laspeyres and Paasche variations, $V_L$ and $V_P$, are observable:[23]

(11) $$V_L(p^0, p^1, q^0, q^1) \equiv p^0 \cdot (q^1 - q^0);$$

(12) $$V_P(p^0, p^1, q^0, q^1) \equiv p^1 \cdot (q^1 - q^0).$$

Hicks (1942) showed that $V_L$ approximates $Q_E$ and $V_P$ approximates $Q_C$ to the accuracy of a first order Taylor series approximation.

As comparisons may be made between periods far apart, value change comparisons are difficult to interpret if the values are not expressed in comparable units. Hence, we recommend using real prices where, for example, the base period's prices are inflated by using the Consumer Price Index (CPI) to put them into comparable units with the current period's prices.[24] Let $\gamma$ denote one plus the CPI growth rate between periods 0 and 1 (which may not be adjacent periods), and $\tilde{p}^0 = \gamma p^0$.[25] Then the observable Bennet (1920) variation or indicator of quantity change $V_B$ is defined as the arithmetic average of the (inflation adjusted) Laspeyres and Paasche variations in (11) and (12):

---

[23] Note that $V_L$ and $V_P$ are difference counterparts to the Laspeyres and Paasche quantity indexes, $Q^L \equiv p^0 \cdot q^1/p^0 \cdot q^0$ and $Q^P \equiv p^1 \cdot q^1/p^1 \cdot q^0$, respectively.

[24] Alternatively, we could deflate current prices to put them into the same units as the earlier period. Having units in a distant past period is, however, typically more difficult to interpret than using current period units. We recommend putting values into comparable units for both welfare and GDP growth adjustments, especially in high inflation environments or if periods are far apart in time. The same approach can be used for spatial comparisons.

[25] We prefer to use the CPI rather than the GDP deflator for adjusting for general inflation, as the GDP deflator behaves perversely if import prices change. This is because the immediate effect of e.g. a fall in import prices is to increase the deflator; see Kohli (1982; 211). Also, Diewert (2002; 556, footnote 14) noted the following: "An example of this anomalous behavior of the GDP deflator just occurred in the advance release of gross domestic product for the third quarter of 2001 for the US national income and product accounts: the chain type price indexes for C, L, X and M decreased (at annual rates) over the previous quarter by 0.4%, 0.2%, 1.4% and 17.4% respectively but yet the overall GDP deflator increased by 2.1 %. Thus there was general deflation in all sectors of the economy but yet the overall GDP deflator increased. This is difficult to explain to the public!"

$$V_B(\tilde{p}^0, p^1, q^0, q^1) \equiv \frac{1}{2}(\tilde{p}^0 + p^1) \cdot (q^1 - q^0)$$

(13)
$$= \tilde{p}^0 \cdot (q^1 - q^0) + \frac{1}{2}(p^1 - \tilde{p}^0) \cdot (q^1 - q^0)$$

$$= V_L(\tilde{p}^0, p^1, q^0, q^1) + \frac{1}{2}\sum_{n=1}^{N}(p_n^1 - \tilde{p}_n^0)(q_n^1 - q_n^0)$$

Thus, the Bennet variation is equal to the Laspeyres variation $V_L$ plus a sum of $N$ Harberger (1971) consumer surplus triangles of the form $(1/2)\ (p_n^1 - \tilde{p}_n^0)(q_n^1 - q_n^0)$.[26]

With certain assumptions on the functional form for the consumer's cost function, the observable Bennet variation can be shown to be *exactly equal* to the arithmetic average of the unobservable equivalent and compensating variations in (9) and (10), respectively.[27]

PROPOSITION 2: An approximation to the Fisher quantity index, $Q^F \equiv [(p^0 \cdot q^1/p^0 \cdot q^0)(p^1 \cdot q^1/p^1 \cdot q^0)]^{0.5}$, can be written as follows, where $V_B$ is the Bennet variation defined in (13):

(14)
$$Q^F \approx \frac{2V_B}{[\tilde{p}^0 \cdot q^0(1 + P^F)]} + 1$$

Note that the numerator in the ratio is two times the Bennet variation, $V_B$. In the context of measuring GDP, equation (14) gives an approximation to a Fisher real GDP index, which is expressed in terms that include a measure of welfare change, $V_B$. We turn now to the form of $V_B$ when there are free goods.

[26] It is also equal to the Paasche variation $V_P$ *minus* a sum of $N$ Harberger triangles.

[27] If the cost function has a flexible, translation-homothetic normalized quadratic functional form, then we have the following exact equality: $V_B = (1/2)Q_E + (1/2)Q_C$. See Proposition 1 in Diewert and Mizobuchi (2009; 353). Normalized prices are needed for this result to be true: "If there is a great deal of general inflation between periods 0 and 1, then the compensating variation will be much larger than the equivalent variation simply due to this general inflation, and an average of these two variations will be difficult to interpret due to the change in the scale of prices. To eliminate the effects of general inflation between the two periods being compared, it will be useful to scale the prices in each period by a fixed basket price index of the form $\alpha \cdot P$, where $\alpha \equiv [\alpha_1, \dots, \alpha_N] > 0_N$ is a nonnegative, nonzero vector of price weights." Diewert and Mizobuchi (2009, 352-353). They recommend choosing $\alpha$ so that a fixed-base Laspeyres price index is used to deflate nominal prices (footnote 34, page 368).

As before, let a new free good be indexed by the subscript 0 and let the $M$ dimensional vectors of period $t$ prices and quantities for the continuing free goods be denoted by $w^t$ and $z^t$ for $t = 0{,}1$. The period 1 quantity of good 0 purchased during period 1 is also observed and is denoted by $z_0^1$. The period 0 reservation price for good 0 is not directly observed but we make an estimate for it, denoted as $w_0^{0*} > 0$. The period 0 quantity is observed and is equal to 0; i.e., $z_0^0 = 0$. Thus the price and quantity data (for the $M+1$ goods) for period 0 is represented by the $1+M$ dimensional vectors $(w_0^{0*}, w^0)$ and $(0, z^0)$ and the price and quantity data for period 1 is represented by the $1+M$ dimensional vectors $(w_0^1, w^1)$ and $(z_0^1, z^1)$.

Welfare change including free goods can be written as follows, where again we adjust period 0 prices by the one plus the growth rate of the CPI between periods 0 and 1, $\gamma$, so that $\tilde{p}^0 = \gamma p^0$, $\widetilde{w}^0 = \gamma w^0$ and $\widetilde{w}_0^{0*} = \gamma w_0^{0*}$:

$$V_B = \tilde{p}^0 \cdot (q^1 - q^0) + \frac{1}{2}(p^1 - \tilde{p}^0) \cdot (q^1 - q^0) + \widetilde{w}^0 \cdot (z^1 - z^0) + \tfrac{1}{2}(w^1 - \widetilde{w}^0) \cdot (z^1 - z^0) + w_0^1 z_0^1 - \tfrac{1}{2}(w_0^1 - \widetilde{w}_0^{0*}) z_0^1. \quad (15)$$

The first term, $\tilde{p}^0 \cdot (q^1 - q^0)$ is simply the change in consumption of market goods valued at the (inflation adjusted) real prices of period 0, a Laspeyres variation as in (11); the second term, $1/2\,(p^1 - \tilde{p}^0) \cdot (q^1 - q^0)$, is the sum of the consumer surplus terms associated with the market goods; the third term, $\widetilde{w}^0 \cdot (z^1 - z^0)$ is the change in consumption of continuing free goods valued at the (inflation adjusted) marginal values to the consumer in period 0; the fourth term, $1/2\,(w^1 - \widetilde{w}^0) \cdot (z^1 - z^0)$, is the sum of consumer surplus terms associated with continuing free goods; the fifth term $w_0^1 z_0^1$ is the value of consumption of the new free good in period 1; and the last term $-\tfrac{1}{2}(w_0^1 - \widetilde{w}_0^{0*}) z_0^1 = \tfrac{1}{2}(\widetilde{w}_0^{0*} - w_0^1) z_0^1$ is the additional consumer surplus contribution of the new free good to overall welfare change.

If the concern is that real GDP omits the contribution from free goods, then we can replace the welfare measure $V_B$ in (14) that uses only market goods with (15), which includes contributions from free goods. Thus, we can adjust real GDP growth, $Q^F$, as follows to reflect the welfare effects of free goods:[28]

$$\text{GDP-B} = Q^F + \frac{2\widetilde{w}^0 \cdot (z^1 - z^0) + (w^1 - \widetilde{w}^0) \cdot (z^1 - z^0) + 2w_0^1 z_0^1}{\tilde{p}^0 \cdot q^0(1 + P^F)} + \frac{(\widetilde{w}_0^{0*} - w_0^1)z_0^1}{\tilde{p}^0 \cdot q^0(1 + P^F)} \tag{16}$$

where the second term on the right hand side of (16) is the contribution from accounting for continuing free goods, and the last term is the adjustment term arising from the entry of a new free good.[29] In an analogous fashion, it can also include an adjustment for new goods. Thus GDP-B denotes GDP growth adjusted for the benefits of new and free goods.[30]

To summarize, GDP-B describes the extension of GDP to incorporate consumer benefits arising from free goods, including free digital goods, which we will value through experiment evidence. Note that our total income method, GDP-$B_T$ of equation (8), extends GDP by including the extra income needed to achieve the same level of utility without the digital goods as with the digital goods. It does not rely on any consumer surplus arguments, only on marginal valuations.[31]

Our second method, GDP-B in equation (16), uses consumer valuations of free digital goods to derive an extension of GDP which is consistent with standard Hicksian concepts of welfare change. Both of these approaches build on standard measures of GDP and are consistent with initiatives, including by national statistical offices, to provide alternative

[28] Welfare change in (15) should also be adjusted for general inflation, especially if inflation is high or if the periods being compared are far apart in time, and similarly for spatial comparisons.
[29] Obviously, (16) can easily be generalized to the case of multiple new regular and free goods.
[30] The "B" in GDP-B can be thought of as standing for the "benefits" arising from new and free goods, or "beyond", as in the literature promoting broader measures of economic wellbeing "beyond GDP".
[31] Schreyer (2022) notes the following: "'Consumer value' is understood as the marginal willingness to pay for or willingness to forego one unit of a particular product – a shadow price, not to be confused with 'consumer surplus' in the sense of a cumulative measure across all consumers' willingness to pay for the utility derived from all the units consumed."

measures of GDP that increasingly encompass welfare from non-market goods and services; see e.g. Hey, Martin and Mkandawire (2019).

Just as our approach makes it possible to calculate GDP-B in a way that accounts for new and free goods, it is straightforward to calculate an alternative measure of labor productivity by simply dividing GDP-B by hours worked. To distinguish it from conventionally-measured productivity, one can label this new metric *Productivity-B.* Other metrics such as total factor productivity-B (TFP-B) can be calculated analogously.

## V. Empirical Examples of GDP-B Applied to Free Digital Goods

In this section we apply our methodology to study the impact of the value to households that is generated by free digital goods. First, we consider the case of Facebook, using online choice experiments to elicit user valuations. Then we consider the valuation of a broader range of digital goods, using laboratory experiments.

To estimate the consumer welfare created by Facebook, we conducted incentive compatible discrete choice experiments on a representative sample of the US internet population. We recruited respondents through an online professional panel provider, Research Now,[32] during the year 2016-17.[33] We targeted consumers that were 18 years or older and lived in the US. We set quotas for gender, age, and US regions to match US census data (File and Ryan 2014) and applied post-stratification for education and household income to obtain our sample. We further asked consumers to select all online services they have used in the last twelve months from a list of 14 options, including a non-existent online service which we used as an attention check. Because our focus is on Facebook users, we disqualified participants who did not use Facebook in the previous twelve months (but we can account for the overall number of Facebook users using secondary data). We also disqualified a small number of users who selected the non-existent service. A total of 2885 participants completed the study.

[32] At the time of data collection, we accessed the panel via Peanut Labs that is part of Research Now, https://www.researchnow.com/. The panel provider complies with industry standards and quality criteria set by, among others, CASRO, MRA and ESOMAR.
[33] These experiments are also reported in Brynjolfsson, Collis and Eggers (2019). In this paper, we combine the studies conducted in summer 2016 and summer 2017 to come up with estimates for the year 2016-17.

In the experiment, each participant was asked to make a single discrete choice between two options: 1) keep access to Facebook or 2) give up Facebook for one month and get paid $E. We allocated participants randomly to one of twelve price points: E = (1, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 1000), i.e., we observe at least 200 participants per price point. Before participants made the decision, we informed them that their decisions were consequential such that we would randomly pick one out of every 200 participants and fulfil that person's selection. We also informed them about how we can monitor their Facebook online status remotely. In order to check if the selected participants gave up Facebook and qualified for the payment, we monitored their online status on Facebook for 30 days.[34] Truth-telling is the optimal strategy for participants in this type of study design (Ding, Grewal and Liechty 2005; Carson, Groves and List 2014).

We fitted a binary logit model to the participants' decisions using the monetary values (in log scale) as predictors. As we intend to obtain an overall valuation of Facebook, we do not add additional control variables to the model.[35] That way, we can estimate the median valuation for our representative sample based on the estimated intercept and monetary parameter. Figure 2 shows the observed shares of participants willing to keep Facebook and the fitted line according to the logit model. According to the model, the median willingness-to-accept (WTA) price for giving up Facebook for one month is $42.17 (bootstrapped 95% confidence interval = [$32.53; 54.47]). This valuation corresponds to the global willingness to accept function in equation (5) of Section 2.

Next, we provide an empirical illustration of the theoretical frameworks for free goods outlined in the previous sections. We use median valuations to avoid the results being unduly influenced by extreme observations. We consider the period from 2003 to 2017; Facebook was founded in 2003-04 and hence became a new free good that year. In our notation of the previous section, 2003 is then period 0 and 2017 is period 1. Assuming a simple linear relationship, the median WTA for Facebook in 2017 ($42.17/month),

[34] It is possible to remotely monitor when someone is last logged in on Facebook for any friend on Facebook.
[35] Estimating a model with control variables shows face validity such that users who spend more time or have more friends on Facebook value it significantly more. Users who also have an Instagram account, a potential substitute, value it significantly less. Gender, age, and income also show significant effects. These extended models could be used for diagnostic purposes, for example, obtaining valuations for segments of users, or to model potentially changing demographics of Facebook users.

translates to \$506.04/year ([390.36; 653.64]).[36] From (5), it is the income in period 1, $m^1$, needed by the median user in compensation for giving up Facebook if the same utility level is to be maintained. We take this to be the representative valuation and normalize the corresponding "quantity" of Facebook for the median user to be $z_0^1 = 1$. Hence, $w_0^1 = m^1/z_0^1 = \$506.04$. Note that this is the price for giving up the 2017 version of Facebook, which includes all its attributes at the time, including the specific content and other participants of the social network, and not simply the intrinsic features of the software.

We also need to determine the reservation price for Facebook in 2003 ($w_0^{0*}$); recall that the reservation price is the price which would induce a utility maximizing potential purchaser of a good to demand zero units of it. Here the good which is having its demand reduced to zero is the 2017 version of Facebook, with all its characteristics at that time, including network size.

Following Hausman (1996), we could consider a reservation price of twice the median WTA, deflated to 2003 dollars; the reservation price for before the 2004 launch of Facebook is then ($w_0^{0*} = 2w_0^1/\gamma \approx$) \$780. This is likely to be a very conservative estimate. Note that the observed demand curve in Figure 2 reflects a much higher reservation price. In fact, there is a significant portion of the sample (>20%) which values Facebook at more than \$1,000 per month. Apple-Cinnamon Cheerios, the product considered by Hausman, can be regarded as quite different to Facebook; it is a new variety of breakfast cereal with plenty of close substitutes, whereas Facebook can be characterized as a novel product.[37] In contrast to the low reservation price from applying Hausman's estimate, the approach of Feenstra (1994) uses a CES framework which requires that all reservation prices are infinity. It seems implausible that prices would have to be so high before demand is reduced to zero.[38]

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[36] Brynjolfsson et al. (2019), find that the relationship between valuation and time period may be non-linear, i.e. valuation for 1 year is a less than 12 times valuation for 1 month. Using hypothetical choice experiments, we find that it is closer to 10 times the valuation for 1 month. Here we assume a linear relationship for simplicity since it is challenging to do a one-year incentive compatible study for Facebook; users can easily set up a new account if asked to forego the use of their account for a whole year. Hence, there is some uncertainty about the appropriate way of converting monthly valuations into annual valuations, with the appropriate way likely to vary across products.

[37] Reinsdorf and Schreyer (2020) note the following regarding the consequences for consumer price inflation of delaying the price measurement of such products: "…novel products may initially exhibit distinctive price change behaviour. The most common pattern is for prices of truly novel products to decline quickly at first, so the bias is upward."

[38] "Thus the CES methodology may overstate the benefits of increases in product availability." Diewert and Feenstra (2022, p. 439).

[ Insert Figure 2 Here ]

Hence, we focus on an alternative approach and estimate the intercept term in a linear regression of WTA on the corresponding share of users who keep Facebook, as plotted in Figure 2; this is the estimate of the monthly WTA that gives a share of zero. Our estimate is from a regression that omits the two extreme observations, for E = $1 and E = $1,000 (p-value = 0.0000, $R^2$=0.88).[39] At extreme values, even a small number of noisy responses will disproportionately affect the reservation price. Multiplying the estimate by twelve yields the 2017 annual reservation price and deflating, using the CPI, yields the reservation price of the 2017 version of Facebook in 2003 dollars. Using this approach, we estimate the reservation price ($w_0^{0*}$) to be $2,152 in 2003 dollars.

Using (15), the estimated contribution to welfare due to Facebook in the U.S. over the period 2003-17 is $231 billion (in 2017 dollars) which translates to $16 billion on average per year.[40] The per user welfare gain over the period 2003-17 is $1,143.

Considering that this is a single new service, this estimate is a substantial contribution to welfare.[41] At the same time, given that the definition of users is that they access their Facebook account via any device at least once per month and the average user is on Facebook for more than 40 minutes per day, or over 240 hours per year,[42] then this estimate does not seem unreasonable.

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[39] We also estimated a regression using all observations. This resulted in a poorer fit (p-value = 0.0038, $R^2$=0.52) and a much higher estimate of the reservation price ($8,126 in 2003$). Using this higher estimate, we would find that the contribution to welfare change over the period 2003-17 is $1,013 billion (in 2017$) which translates to an average of $72 billion per year. Per user, the welfare change over the period 2003-17 is $5,018 which translates to $358.48 on average per year.

[40] Notes:
$w_0^1$ = $506.04 (95% C.I.: [390.36; 653.64])
$\gamma$ = 1 + Growth rate of CPI = 1.3
Number of Facebook users in US in 2017 = 202 million
Data sources:
Chained CPI-All Urban Consumers, not seasonally adjusted, index for December 2003 to December 2017 is 1.2975, or 29.75%.
https://www.bls.gov/cpi/data.htm
Internet users who access their Facebook account via any device at least once per month.
https://www.statista.com/statistics/408971/number-of-us-facebook-users/

[41] Note that we are not accumulating benefits from the years in between 2003 and 2017. We are simply comparing the welfare change between two periods: 2003 when Facebook did not exist and 2017 when the 2017 version existed. The comparison between these two years, as opposed to any of the intervening years, is of interest as there was no close substitute to any subsequent version of Facebook in 2003. In the intervening years, if each version of Facebook, with increasing network size, is treated as a new good then we would need to also model the impact of the exiting versions of Facebook. We do not have the valuations required to do such a study.

[42] See https://www.emarketer.com/Chart/Average-Time-Spent-per-Day-with-Facebook-Instagram-Snapchat-by-US-Adult-Users-of-Each-Platform-2014-2019-minutes/211521

Next, we turn to GDP-B growth to get an idea of the change that would result from extending the usual definition of GDP to include a free service such as Facebook. First, we consider the total income approach of equation (8) in Section 3. We need the total nominal income ($T$) for both 2003 and 2017, which we calculate as follows:

$T^0 =$ nominal GDP in 2003 + $m^0$ x No. of Facebook users in US in 2003
= \$11.51 trillion + 0 ≈ \$11.51 trillion
$T^1 =$ nominal GDP in 2017 + $m^1$ x No. of Facebook users in US in 2017
= \$19.39 trillion + \$506.04 x 202 million
≈ \$19.49 trillion.

That is, total nominal income using GDP-$B_T$ is higher by \$102 billion in 2017 since the value of Facebook to consumers is taken into account.[43] Recall, from Section 3, that this can be interpreted as the amount that consumers in aggregate would need in compensation in order to attain the same level of utility if access to Facebook had foregone in 2017. This is for the 2017 version of Facebook, including all its characteristics, such as the content and participants of the network. Hence, the result is independent of the other changes in the characteristics of Facebook over the intervening years since its launch.

From equation (8), in our case GDP-$B_T$ $= (T^1/T^0)/P^F = (19.49/11.51)/1.31 \approx 1.295$. Thus GDP-B grew by 29.50% between 2003 and 2017 using the total income approach, whereas conventionally-measured real GDP grew by 28.82%, giving a percentage point difference of 0.68 over the entire period, or 0.05 per year on average.[44]

[43] A referee suggests a quick calculation to gauge whether one might expect a number as large as \$102 billion. This follows the idea of Schreyer (2022) of measuring own account leisure activity. If the average U.S. wage is \$20 per hour, with 202 million users of Facebook who use it 240 hours per year, then the average annual valuation for each user is \$4,800 per year, with a total valuation of almost \$970 billion. By comparison, our estimate seems conservative.
[44] Recall that this can be thought of as an underestimate of the additional growth from using GDP-B, as the deflator is not adjusted for the impact of new goods prices.

Now we consider our second approach. From the last line of equation (16) of Section 4, we have the following:

Adjustment to real GDP-B growth from accounting for Facebook over 2003-2017

$$= \frac{(\widetilde{w}_0^{0*} - w_0^1)z_0^1}{\tilde{p}^0 \cdot q^0(1+P^F)} \times \text{No. of Facebook users in US in 2017}$$

$$= \frac{(\gamma w_0^{0*} - w_0^1) \times \text{No. of Facebook users in US in 2017}}{\gamma(\text{Nominal GDP in 2003})(1 + \text{GDP deflator}/\gamma)}$$

where the quantity for the median user is normalized to 1, $z_0^1 = 1$, and $P^F \equiv [(p^1 \cdot q^0 / \tilde{p}^0 \cdot q^0)(p^1 \cdot q^1/\tilde{p}^0 \cdot q^1)]^{0.5} = GDP\ deflator/\gamma$ as $\tilde{p}^0 = \gamma p^0$. The GDP adjustment is a lower bound on the amount to add to GDP-B growth using this approach because we use official estimates of the CPI ($\gamma$) and the GDP deflator (which are unadjusted for the introduction of new goods) in the denominator. Normally, both price indexes would be lower if we account for the fact that the price of the new goods typically fall following their introduction.[45]

From Table 1, for the reservation price of $w_0^{0*} = \$2,152$ in 2003, accounting for Facebook would increase real GDP-B growth by 1.54 percentage points from 2003 to 2017 (or, using the 95% CI estimates of $w_0^1$: [1.44, 1.62]). In other words, this amounts to an increase in real GDP-B growth of 0.11 percentage points on average per year over this period. Real GDP grew by 28.82% and real GDP-B grew by 29.16% including the contribution from Facebook. Average real GDP growth over this period was 1.83% per year. Adding the contribution of Facebook means that GDP-B grew by 1.91% per year.[46]

An identical analysis indicates that productivity-B also grew by an average of 0.11 percentage points more each year than conventional productivity. In comparison, total US GDP grew by 2.4% in 2017, and labor productivity grew 1.2%. The additional 0.11% for

[45] See Diewert, Fox and Schreyer (2018) and Reinsdorf and Schreyer (2020).

[46] The corresponding growth estimate from using the reservation price estimated using all observations ($8,126) is 2.20% per year on average. As noted, following Hausman (1996) the reservation price is ($w_0^{0*} = 2w_0^1/\gamma \approx$) $780, which yields a growth estimate of 1.84% per year on average. At least in this Facebook example, from Table 1 we see that GDP-B (1.84%) and GDP-B$_T$ (1.87%) yields similar growth estimates in this case.

just one product, Facebook, is significant, suggesting that the benefits from free goods, as measured by GDP-B and productivity-B should not be ignored when assessing economic growth.

[ Insert Table 1 Here ]

Our methods can readily be applied to a variety of other digital goods. In the Appendix, we present results of a series of laboratory experiments evaluating GDP-B contribution of seven other digital goods (Instagram, Snapchat, Skype, WhatsApp, Digital Maps, LinkedIn and Twitter), in addition to Facebook.

## VI. Applying GDP-B to Adjusting for New Features in Smartphone Cameras

Smartphones have added numerous features since their introduction, often absorbing the functionality of other devices. For instance, smartphone cameras are now the primary devices for taking photos. From the 1997 to 2017, the dominant photographic technology shifted from analog cameras to digital cameras to smartphone cameras. The total number of digital cameras shipped worldwide dropped from 121 million units in 2010 to 24 million units in 2016,[47] while worldwide smartphone sales increased from 297 million in 2010[48] to 1.5 billion in 2016.[49] Moreover, the marginal cost of taking a photo has fallen to approximately zero with smartphones, compared with up to 50 cents per photo for developing film and printing photos in the analog era. Just between 2010 and 2017, the number of photos taken worldwide increased from 350 billion to an estimated 2.5 trillion.[50] Furthermore, a photo taken on a smartphone today is in many ways superior to a photo taken on an average camera twenty years ago, including its ability to be easily edited, stored, shared or repurposed far more easily.

[47] http://www.cipa.jp/stats/dc_e.html
[48] http://www.gartner.com/newsroom/id/1543014
[49] http://www.gartner.com/newsroom/id/3609817
[50] https://www.nytimes.com/2015/07/23/arts/international/photos-photos-everywhere.html

To illustrate the problem this change creates, as a motivating example we consider a simple case of two goods, each available in two periods: a digital camera and a feature phone[51] in period 0, and a smartphone with a digital camera in period 1.[52] Suppose that the value of the camera to the consumer is $v_c$, the value of the simple feature phone is $v_f$, and the value of the smartphone is $v_c + v_f$. Assume that a device fully depreciates in a time period, i.e., a consumer has to purchase new devices each period. Also assume that a consumer buys both the camera and the feature phone in period 0 and only the smartphone in period 1, and there are a total of $x$ such consumers. Suppose that the price of the camera is $p_c$ in period 0, the price of the feature phone is $p_f$ in period 0, and the price of the smartphone is also $p_f$ in period 1. Then we have the following consumer surplus measures, $CS^0$ and $CS^1$, for periods 0 and 1, respectively:

$$CS^0 = (v_c - p_c)x + \left(v_f - p_f\right)x \geq 0,$$
$$CS^1 = \left(v_c + v_f - p_f\right)x \geq 0.$$

Then the change in consumer surplus between periods 0 and 1 is $CS^1 - CS^0 = p_c x$. This is the cost saving of not buying the digital camera in period 1 because its functionality is now included in the smartphone. However, the contribution of these goods towards conventionally-measured GDP (i.e., the market price of final goods) is $(p_c + p_f)x$ in period 0 but only $p_f x$ in period 1. Hence the change in conventionally-measured GDP from period 0 to period 1 is $-p_c x$, which is exactly the opposite of the change in consumer surplus. Therefore, while conventionally-measured GDP goes *down* due to people not purchasing the digital camera, consumer surplus and GDP-B go *up*. The measured decrease in conventional GDP occurs because, even though it has the same market price ($p_f$) as the feature phone in this example, the smartphone is a higher quality product. That is, there is an implicit fall in price in shifting from the feature phone to the smartphone which is not being captured.

[51] A feature phone is a phone defined as a phone with no camera for the purposes of this example.
[52] We thank Hal Varian for sharing his notes on GDP and welfare which contained a version of this example.

To accurately reflect welfare changes, it is clear that national statistics should account for quality improvements in smartphones, including the introduction and improvements in smartphone cameras. Until January 2018, the BLS only incorporated quality adjustments for data plans offered by mobile network operators in the CPI.[53] Starting from January 2018, there is now quality adjustment of the CPI for telephone hardware, calculators and other consumer information items using hedonic modelling of the value of characteristics;[54] this is used by the Bureau of Economic Analysis (BEA) to deflate Personal Consumption Expenditures for telephone and facsimile equipment in constructing real GDP; see BEA (2014, Chapter 5, Table 5.A). Therefore, even though GDP statistics capture paid goods such as smartphones, they have failed for many years to completely capture quality adjustments in the US. Furthermore, many countries still do not make any quality adjustments for smartphones. Some of those that do (e.g. UK, New Zealand and Germany) have only begun doing so recently; see e.g. Wells and Restieaux (2014, Table 1). Even when they do attempt to adjust for quality improvements, Groshen et al. (2017) state that hedonic techniques are not suitable for products such as smartphones when the set of relevant characteristics frequently change, Byrne (2019) shows that prices do not necessarily begin to fall faster, and the results of Cole *et al*. (1986) suggest that an incumbent with market power can prevent the price from falling to match the (quality-adjusted) price of the new good.[55] Note that quality improvements, such as the addition of a camera feature to a smartphone, can also be thought of as additions of new goods as described in our framework.

[53] https://www.bls.gov/cpi/factsheets/telephone-services.htm.

[54] The methodology and characteristics used for the hedonic modelling are currently not published. https://www.bls.gov/cpi/factsheets/telephone-hardware.htm. Aizcorbe, Byrne and Sichel (2020) also developed hedonic indexes for smartphones.

[55] If we consider software features (including operating system and various apps) as part of the set of relevant characteristics for hedonic quality adjustments, then it is impossible to perform hedonic modelling because firms do A/B testing continuously and seek to improve these features as frequently as daily.

To demonstrate the importance of quality change as can be captured by GDP-B, we elicit the value generated of smartphone cameras for participants in a university laboratory in the Netherlands and compare that with the approximate price paid for them.

Specifically, we applied an incentive compatible BDM lottery (Becker, DeGroot, and Marschak 1964) in order to estimate the consumers' valuation of their smartphone camera. We asked participants to state the minimum amount of money they would request in order to give up their smartphone camera (both main camera and front camera) for one month. Participants were informed that this amount would serve as a bid in a lottery. If their minimum bid to forego their camera would be higher than a random price, drawn from a uniform distribution, they could keep access to their smartphone camera but would not receive any cash. If the random price exceeded their minimum requested amount, they would be paid the random price, provided that they would give up using the smartphone camera for one month. The utility-maximizing strategy of the participants in the BDM lottery is to provide a bid that matches their true valuation. Accordingly, we use the bids as measures of WTA to give up smartphone cameras.

In order to induce incentive compatibility and make the answers consequential, we provided further information that one out of fifty participants would be selected for the lottery and that we would block their smartphone cameras with a special sealing tape if their bid was successful (see Figure A2). The sealing tape would break if the participants tried to peel it off so that it was not possible to re-apply it. We also signed the tape so that it was not possible to buy the same type of seal and re-apply a seal. If, after the one-month period, the original seal was still intact participants were rewarded with the money and the seal could be removed.

The study was conducted in the laboratory of a large Dutch university in November/December 2017 (to not cover the holiday season, respondents were allowed to postpone giving up their camera until January 2018). In total, 213 students participated.

The sample was relatively balanced in terms of gender (54.5% were female) and represented the student population in terms of age (87.8% were between 18 and 24 years old). Participants reported that they use their smartphone cameras frequently and take, on average, 21.7 pictures (median = 10) and 2.3 videos (median = 1) per week. For 59% of the participants the smartphone camera is the only camera they possess. Only 16.4% own a separate point-and-shoot camera, and 18.8% a DSLR camera.

Directly eliciting monetary values in a survey leads to the observation of price thresholds, i.e., certain values that are stated more frequently. In our results, we observe that the bids 40, 50, 100, 150, 200 were each entered by more than 5% of the sample. The median bid that was given for the smartphone camera was €100. However, this median bid does not account for the price thresholds in the demand function. For example, the bids imply that 41% of the students would not give up their smartphone camera for €100, but 54% would at €100.01. To smooth the demand function, we therefore fitted a (multiplicative) function to the observed shares of students willing to accept the offer. This function explains 87.7% of the variation in demand and is depicted in Figure 3.

According to the fitted values, the median WTA for giving up the smartphone camera for one month is €68.13, albeit having a wide confidence interval (95%-CI = [€33.53; €136.78]). This implies a median annual WTA of over €800 for smartphone cameras, at least for the students in our sample.

[ Insert Figure 3 Here ]

Analysts have estimated that it costs $20-$35 to manufacture the smartphone cameras present in current flagship models.[56] Similarly, a modular smartphone sold in the Netherlands can add front and back cameras for an additional charge to consumers of €70.[57] This study provides strong evidence that consumers obtain a significant amount of surplus from using their smartphone cameras and this surplus is an order of magnitude larger than what they actually pay.[58] Hence, there has been a large implicit price decline arising from quality change; the services received from the smartphone have increased due to quality change but this is not reflected in the measured price. Therefore, even for paid goods such

[56] E.g. http://www.techinsights.com/about-techinsights/overview/blog/cost-comparison-huawei-mate-10-iphone-8-samsung-galaxy-s8/, https://technology.ihs.com/595738/ihs-markit-teardown-reveals-what-higher-apple-iphone-8-plus-cost-actually-buys
[57] https://shop.fairphone.com/en/spare-parts (accessed January 2018)
[58] As expected, in a competitive market, most of the benefits from innovation go to consumers, not producers (Nordhaus, 2004)

as smartphones, it is crucial to adjust for quality improvements before estimating GDP statistics. This might not be an issue if consumers derived an equally large surplus from what they actually paid for while using digital or analog cameras previously. However, it is hard to reconcile this hypothesis with advancements in smartphone cameras and the reduction in costs of taking photos.

We use our total income approach for GDP-B in equation (8), which does not require calculation of a reservation price for the good in the period before it appears, to calculate an estimate of the contribution of accounting for value of the smartphone camera to consumers;[59] we estimate an average contribution of 0.21 percentage points per year to GDP-$B_T$.[60]

## VII. Conclusion

Welfare is central to economics, yet poorly measured in our national accounts. Contributions to welfare are especially badly measured in two areas where policy choices are especially consequential: new goods and digital goods. This paper develops a framework for accurately measuring the impacts of new and free goods on welfare. The result is a new measure, GDP-B and corresponding productivity metrics. These measures reveal and quantify the mismeasurement that arises from not fully accounting for the consumption of goods which are new, free or both new and free. This is of increasing

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[59] Although Dutch university students do not provide a representative sample of the population, our results are indicative of the approximate size of the adjustment term to real GDP-B growth. Having a small and unrepresentative sample, we refrain from attempting to estimate reservation prices and only use the total income approach in this context.

[60] This is the arithmetic percentage point difference between the growth in GDP-$B_T$ and official real GDP growth. It is calculated by assuming the following: (i) Smartphones with cameras appeared from July 2008, the date of the launch of the first iPhone in the Netherlands. Period 0 is then taken to be Q4 of 2008. (ii) Based on EuroStat survey information on individuals who used a mobile or smartphone to access the internet (https://www.cbs.nl/en-gb/news/2018/05/the-netherlands-leads-europe-in-internet-access), the number of users of smartphones in 2017 was estimated to be 84% of the population of the Netherlands of age 15 and above (constituting 83.6% of the population). With a total population of 17 million this translates to approximately 12 million users in 2017. (iii) The quarterly median WTA is €272.52 (monthly WTA of €68.13 times 4), and this is taken as the appropriate price for valuing the smartphone cameras when using quarterly data; the purchase price of the camera component of the phone is assumed to be very small, so is treated as approximating zero for simplicity. With these assumptions, total income can be calculated for Q4 of 2017 as the quarter's nominal GDP (https://fred.stlouisfed.org/series/CPMNACNSAB1GQNL, accessed 22 January 2019) plus the value of the smartphone cameras. The total income quantity index between quarter 4 of 2008 and Q4 of 2017 can then be calculated by deflating by the official quarterly GDP deflator, and the difference with official real GDP growth (https://fred.stlouisfed.org/series/CLVMNACNSAB1GQNL, accessed 22 January 2019) from Q4 2008 to Q4 2017 calculated: 1.114-1.095=0.019. That is, the difference with official real GDP is almost 2 percentage points over the nine years, or an arithmetic average of 0.21 percentage points per year.

relevance in the modern digital economy given the frequent introduction of new goods and growing presence of zero-priced goods.

We also show how to use the GDP-B concept to derive an estimate of the addition to real GDP growth that would be required to account for the introduction of a new free good without having to recalculate GDP numbers published by national statistical offices. Our total income approach does not rely on consumer surplus arguments, and can approximate a true index of GDP that is calculated using reservation prices to value new free goods.

Appropriately, we freely drew on both old and new literatures to define a framework for measuring welfare change. We were able to use this framework to derive an explicit term that is the marginal value of a new free good on welfare change. That is, we get a measure of the contribution to welfare of a new good, and hence the extent of welfare change mismeasurement if it is omitted from statistical agency collections that rely on conventional measures of GDP and productivity. Accounting for the consumption of new and free goods in GDP gives us a new metric, GDP-B, which expands GDP and the national accounts beyond the traditional definitions, and a set of companion metrics like Productivity-B.

We derive two empirical implementations. One requires the estimation of reservation prices, while the other, based on the concept of "total income" avoids this necessity. Hence, we derive explicit adjustments for both welfare change and equivalent real GDP growth that account for new and free goods, both of which are new to the literature.

We propose a way of implementing these adjustments using incentive-compatible discrete choice experiments. We use this approach to quantify the GDP-B adjustment for the case of an important new and free good, Facebook, using a representative sample of the US internet population. We provide two estimates for the impact of incorporating Facebook into GDP-B, ranging from 0.05 to 0.11 percentage points per year on average from 2004. Since GDP is the numerator used to calculate both labor productivity and total factor productivity, both of these numbers would change by the same amount per year when accounting for new and free goods using GDP-B. These are significant effects, especially considering that Facebook is just one product. A more comprehensive application of our approach would undoubtedly add to these estimates. Indeed, using laboratory experiments in the Netherlands, we find that the additions to GDP-B generated by many other digital goods is also quite large.

Using another laboratory experiment for computing the welfare created by smartphone cameras, we also show how these methods can account for new features in smartphones and other products, thereby better capturing the value of rapid quality change and new features. To elicit the consumer valuations of quality attributes, the experimental approach proposed and applied here is to block certain features of the goods (e.g. cameras in smartphones), or even take away the entire good, in exchange for monetary compensation. This is a practical alternative way to estimate the valuations of product characteristics for adjusting price indexes, as opposed to hedonic techniques, especially when the set of characteristics of goods changes rapidly. Our approach quantifies the enormity of the contribution from the new features made available and widely adopted in smartphones. They added an average of 0.62 percentage points per year to GDP-$B_{T.}$

Our experiments uncovered high valuations for networked goods like Facebook that were missed by conventional approaches. This raises a host of interesting questions that can be explored in further. In future work, it would be insightful to delve deeper into these individual apps and study the sources of these valuations and the year-on-year changes in valuations. In addition to product quality, network effects and focal point effects are also contributing factors towards these valuations. Furthermore, many of these digital goods are also associated with externalities and a parallel stream of research is needed to explore these issues in greater detail; for example, Allcott et al. (2020) explore the impact of Facebook on subjective well-being, news consumption and political polarization.[61]

The techniques and framework introduced in this paper are applicable not only to digital goods like Facebook and digital maps, and new goods like smartphone cameras and space tourism and, but also conventional goods as well, from breakfast cereal to jet travel, some of which have significantly higher or lower contributions to welfare than might be inferred from expenditures alone. Furthermore, there are opportunities to extend the approach to

[61] It is also possible to pose speculative questions about how much better off consumers would be if some digital goods had never been invented. Indeed, such an exercise is possible for conventional goods as well.

non-market goods as well, like government mandates and Covid tests, ultimately providing a more comprehensive and meaningful measure of welfare changes.

In conclusion, GDP-B and the related metrics proposed in this paper enable a more thorough exploration of the impacts of new and free goods on welfare, with significant potential policy implications. Not only can these metrics help us understand the true magnitudes of welfare changes over time, but they can also clarify which goods and innovations are actually contributing the most to economic growth and well-being as the economy evolves.

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TABLE 1— GDP-B CONTRIBUTIONS, FACEBOOK

| | Total Income | Reservation Price |
|---|---|---|
| Reservation Price $w_0^{0*}$, 2003$ | — | $2,152 |
| Percentage Points, 2003-2017 | 0.68 | 1.54 |
| Per year | 0.05 | 0.11 |
| GDP-B Growth per year without Facebook (i.e. GDP growth) | 1.83 | 1.83 |
| GDP-B Growth per year with Facebook | 1.87 | 1.91 |

*Notes:* $w_0^1 =$ \$506.04 (95% C.I.: [390.36; 653.64]), $\gamma$ = 1 + Growth rate of CPI = 1.3, GDP Deflator[62] = 1.31, $P^F = \text{GDP deflator}/\gamma = 1.0078$, Number of Facebook users in US in 2017 = 202 million, Nominal GDP for 2003[63] = \$11.5 trillion; The reservation price is 12 times the intercept from a linear regression of monthly WTA on the corresponding share of users who keep Facebook, dropping the observations for the two extreme observations, E=\$1 and E=\$1000 (p-value = 0.0000, $R^2$=0.88). "Per year" estimates are calculated using the arithmetic mean of the percentage point difference over the period. "Growth per year" estimates are calculated using geometric means.

[62] GDP Implicit Price Deflator, annual, not seasonally adjusted, 2010=100: Growth for 2003 to 2017 = 112.05/85.69 = 1.31. https://fred.stlouisfed.org/series/USAGDPDEFAISMEI
[63] Gross Domestic Product, annual, not seasonally adjusted: https://fred.stlouisfed.org/series/GDPA. The beginning of year value for a 2004 product launch is the GDP of 2003.

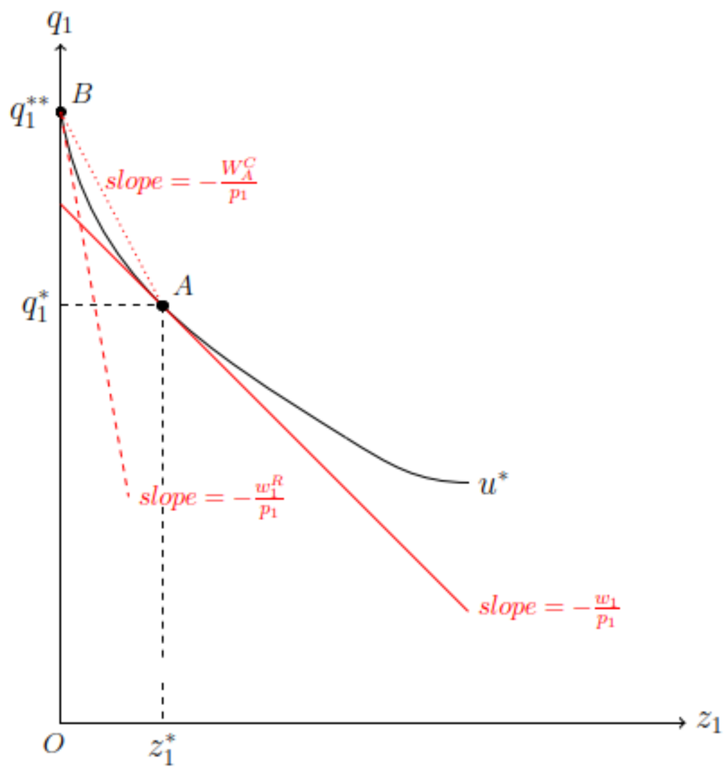


FIGURE 1a. TWO COMMODITY CASE, $w_1 > 0$

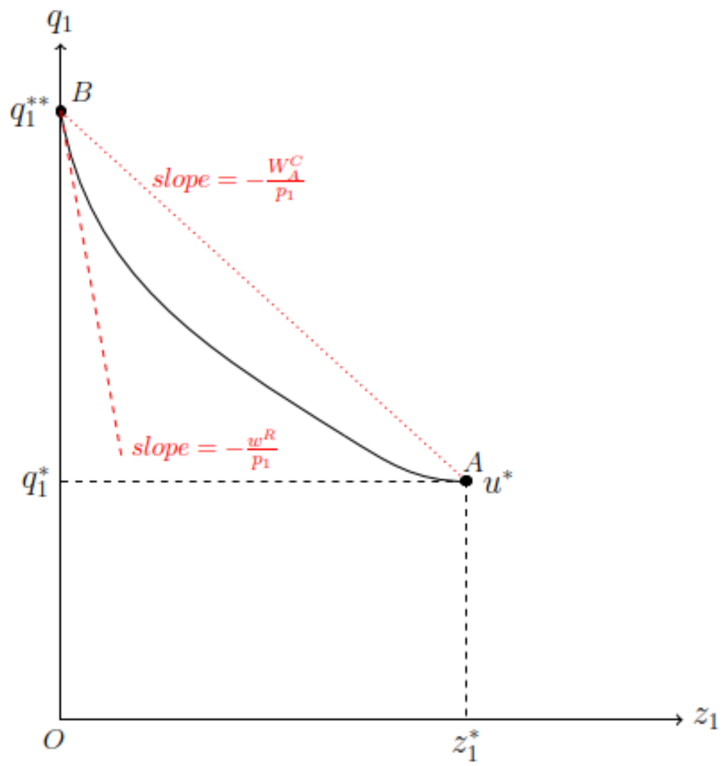


FIGURE 1b. TWO COMMODITY CASE, $w_1 = 0$

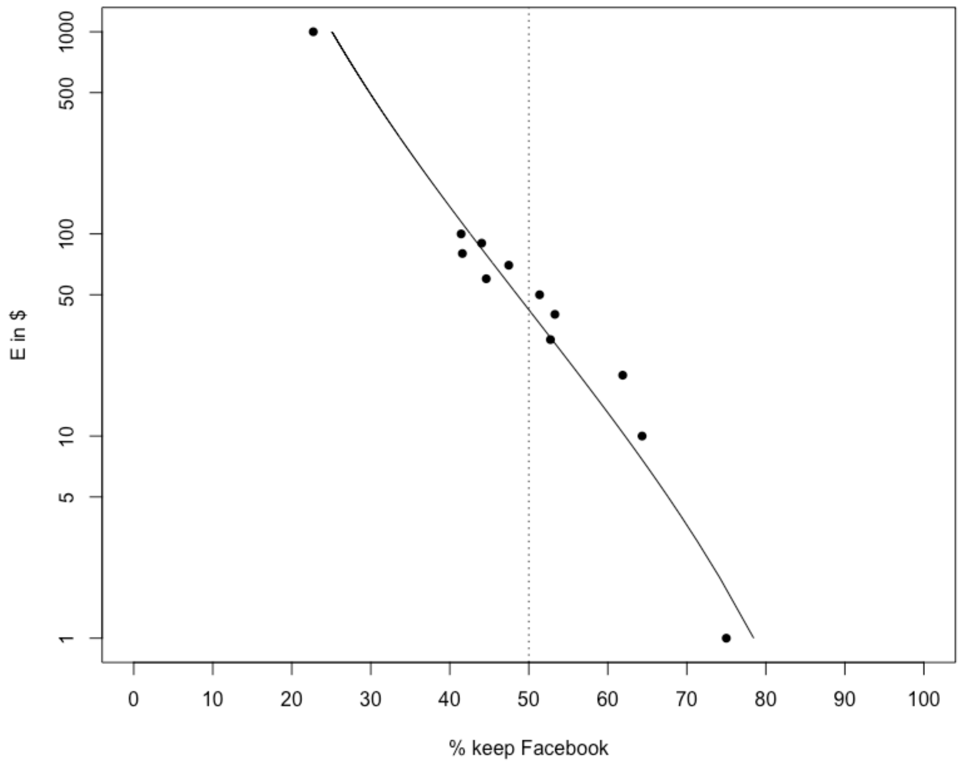


FIGURE 2. WTA DEMAND CURVE FOR FACEBOOK

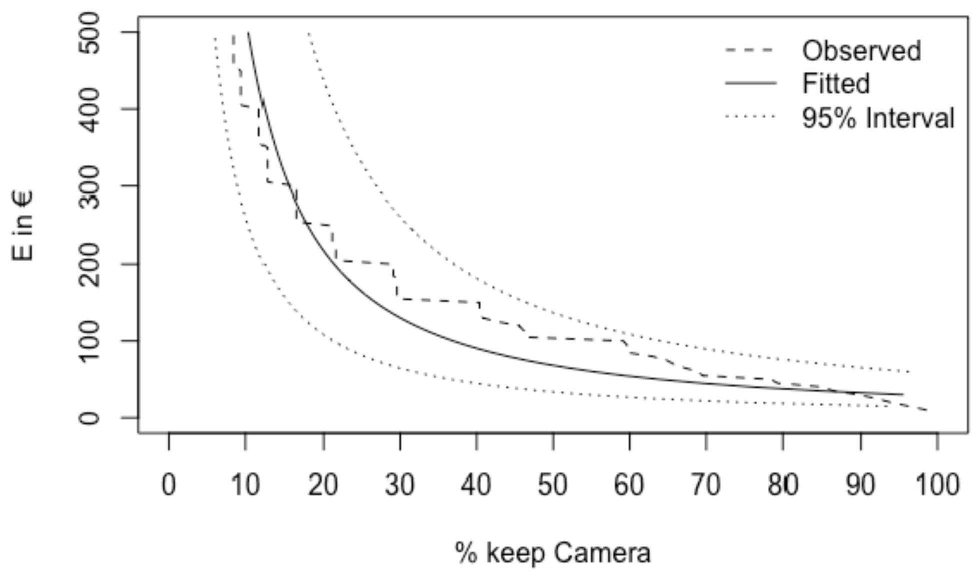


FIGURE 3: DEMAND FUNCTION FOR THE SMARTPHONE CAMERA

## APPENDIX

*Proof of Proposition 1:*

Consider the case of a new free good that is available in period 1 but not in period 0, denoted $z_0^1$. We assume that the observed quantities $(q^t, z_0^t)$ are the solution to the regular cost minimization problem (3) and $q^t$ is the solution to the conditional cost minimization problem in (2), for periods $t = 0{,}1$.

The maximum overlap Fisher quantity index between periods 0 and 1, $Q_{MO}^F$, is

(A1) $$Q_{MO}^F \equiv \left\{\frac{p^0 \cdot q^1}{p^0 \cdot q^0} \cdot \frac{p^1 \cdot q^1}{p^1 \cdot q^0}\right\}^{\frac{1}{2}}$$

with a corresponding maximum overlap price deflator, $P_{MO}^F$:

(A2) $$P_{MO}^F \equiv \left\{\frac{p^1 \cdot q^0}{p^0 \cdot q^0} \cdot \frac{p^1 \cdot q^1}{p^0 \cdot q^1}\right\}^{\frac{1}{2}}$$

Using $P_{MO}^F$ in (A2) as the GDP deflator, GDP-B$_\text{T}$ from (8) can be written as follows:

(A3) $$\begin{aligned}\text{GDP-B}_\text{T} &= \frac{p^1 \cdot q^1 + m^1}{p^0 \cdot q^0} \cdot \frac{1}{P_{MO}^F} \\ &= Q_{MO}^F + \frac{m^1}{p^0 \cdot q^0} \cdot \frac{1}{P_{MO}^F}\end{aligned}$$

using the fact that the Fisher index satisfies the product test from the axiomatic approach to index numbers, so that $Q_{MO}^F = (p^1 \cdot q^1 / p^0 \cdot q^0)/P_{MO}^F$.

The ratio of GDP-B$_\text{T}$ to the Fisher maximum overlap index in (A1) is

(A4) $$\frac{\text{GDP-B}_\text{T}}{Q_{MO}^F} = 1 + \frac{m^1}{p^1 \cdot q^1}$$

As $m^1$, the extra income households would need in compensation in period 1 for the loss of the free good is positive, the right-hand side of (A4) gives the positive amount by which the maximum overlap Fisher quantity index needs to be multiplied by to yield GDP-B$_T$.

In contrast to the maximum overlap quantity index in (A1), the true Fisher quantity index, $Q_T^F$, can be written as follows:

(A5)
$$Q_T^F \equiv \left\{\frac{p^0 \cdot q^1 + w_0^{0*} z_0^1}{p^0 \cdot q^0} \cdot \frac{p^1 \cdot q^1}{p^1 \cdot q^0}\right\}^{\frac{1}{2}}$$

where $w_0^{0*} > 0$ denotes the reservation price for the new free good that drives its demand to zero units in period 0, so that $z_0^0 = 0$, and where the free good is consumed in positive quantities in period 1, so that $z_0^1 > 0$.

Using a first order Taylor series approximation, the ratio of the true Fisher index to the maximum overlap index can be written as follows:[64]

(A6)
$$\frac{Q_T^F}{Q_{MO}^F} = \left[1 + \frac{w_0^{0*} z_0^1}{p^0 \cdot q^1}\right]^{\frac{1}{2}}$$
$$\approx 1 + \frac{1}{2}\frac{w_0^{0*} z_0^1}{p^0 \cdot q^1} = 1 + \frac{1}{2}\frac{w_0^{0*} z_0^1}{p^0 \cdot q^0}\ P^P$$

where $P^P = p^1 \cdot q^1 / p^1 \ \cdot q^0$ is a Paasche price index. If, for example, the estimated period 1 reservation price is exactly twice the average compensation per unit of $z_0$, following the approximation of Hausman (1996, p. 210), then using $P^P$ as the general rate of inflation, the period 0 reservation price is $w_0^{0*} = 2(m^1/z_0^1)/P^P$, and (A6) becomes

[64] A first order Taylor series approximation to $g(x) \equiv (1 + x)^{\frac{1}{2}}$ around $x = 0$ is $1 + (1/2)x$.

(A7) $$\frac{Q_T^F}{Q_{MO}^F} \approx 1 + \frac{m^1}{p^1 \cdot q^1}$$

Comparing (A7) with (A4), we see that the right-hand sides are equivalent. Hence, GDP-$B_T$ will approximate the true Fisher quantity index $Q_T^F$ for reservation prices $w_0^{0*} = 2(m^1/z_0^1)/P^P$.

Alternatively, if homothetic preferences are assumed, then an estimate of the reservation price can be derived as $w_0^{0*} = 2(m^1/z_0^1)/P_{MO}^F$.[65] In this case, (A6) becomes

(A8) $$\frac{Q_T^F}{Q_{MO}^F} \approx 1 + \frac{m^1}{p^1 \cdot q^1}\frac{P^P}{P_{MO}^F}$$

and comparing (A8) with (A4), GDP-$B_T$ will approximate the true Fisher quantity index $Q_T^F$ if $P^P$ is (approximately) equal to $P_{MO}^F$.

Hence, for a range of estimates of the reservation price for the free good, GDP-$B_T$ will approximate the true Fisher quantity index of GDP. ■

*Proof of Proposition 2:*

Value change can be decomposed into Bennet quantity and price variations, as follows:

(A9) $$p^1 \cdot q^1 - \tilde{p}^0 \cdot q^0 = V_B(\tilde{p}^0, p^1, q^0, q^1) + I_B(\tilde{p}^0, p^1, q^0, q^1),$$

where $V_B(\tilde{p}^0, p^1, q^0, q^1) \equiv 1/2\,(\tilde{p}^0 + p^1) \cdot (q^1 - q^0)$ and $I_B(\tilde{p}^0, p^1, q^0, q^1) \equiv 1/2\,(q^0 + q^1) \cdot (p^1 - \tilde{p}^0)$. Equation (A8) can thus provide a decomposition into quantity and price components for any value change, including a change in nominal GDP.

Value change can also be expressed as follows, where $P$ and $Q$ are price and quantity indexes, respectively, that satisfy $P \times Q = p^1 \cdot q^1 / (\tilde{p}^0 \cdot q^0)$:[66]

[65] See Diewert, Fox and Schreyer (2022), and Brynjolfsson, Collis, Diewert, Eggers and Fox (2020).

[66] That is, the formulae for the indexes $P$ and $Q$ are such that the product test from the axiomatic approach to index numbers is satisfied. The expression in (A9) draws on Diewert (2005; 335).

$$p^1 \cdot q^1 - \tilde{p}^0 \cdot q^0 = \tilde{p}^0 \cdot q^0 \left[\frac{p^1 \cdot q^1}{\tilde{p}^0 \cdot q^0} - 1\right] = \tilde{p}^0 \cdot q^0 [PQ - 1]$$

$$\text{(A10)} \qquad = \frac{1}{2}\tilde{p}^0 \cdot q^0 [2PQ - 2]$$

$$= \frac{1}{2}\tilde{p}^0 \cdot q^0 [(1 + P)(Q - 1) + (1 + Q)(P - 1)]$$

$$= V_E + I_E$$

where $V_E = (1/2)\,\tilde{p}^0 \cdot q^0(1 + P)(Q - 1)$ is a quantity change indicator and $I_E = (1/2)\,\tilde{p}^0 \cdot q^0(1 + Q)(P - 1)$ is a price change indicator. If $P$ and $Q$ are replaced by superlative indexes,[67] such as the Fisher or Törnqvist, then the resulting indicators can also be called superlative.

A corollary of Proposition 9 of Diewert (2005; 338) is that the Bennet indicator of quantity change approximates any superlative indicator to the second order at any point where the two quantity vectors are equal and where the two price vectors are equal. Consider the following expression for the Fisher superlative quantity change indicator, $V_E^F$:

$$\text{(A11)} \qquad V_E^F \equiv \frac{1}{2}\tilde{p}^0 \cdot q^0 (1 + P^F)(Q^F - 1) \approx \frac{1}{2}(\tilde{p}^0 + p^1) \cdot (q^1 - q^0) = V_B,$$

where $P^F \equiv [(p^1 \cdot q^0 / \tilde{p}^0 \cdot q^0)(p^1 \cdot q^1 / \tilde{p}^0 \cdot q^1)\,]^{0.5}$ is a Fisher price index and $Q^F \equiv [(p^0 \cdot q^1 / p^0 \cdot q^0)(p^1 \cdot q^1 / p^1 \cdot q^0)\,]^{0.5}$ is the Fisher quantity index, or real GDP growth in our context, and $V_B$ is the Bennet quantity indicator.[68] Recall that the Bennet indicator of quantity change is the symmetric arithmetic average of first-order approximations to the Hicksian equivalent and compensating variations of equations (9) and (10).[69] Hence, the

[67] See Diewert (1976) on superlative index numbers.
[68] See Diewert (2005). If real GDP growth is not constructed using a superlative index such as the Fisher, but rather using e.g. a Laspeyres index as is standard in many countries, there will still be an approximation as in (16), but it may not be as accurate.
[69] Alternatively, under the Diewert and Mizobuchi (2009) assumptions on the functional form for the consumer's cost function, the Bennet indicator is *exactly* equal to the arithmetic average of the equivalent and compensating variations.

Fisher superlative quantity change indicator, $V_E^F$ in (A10), can be interpreted as an approximation to a welfare change indicator, $V_B$.

Re-arranging (A10), we get an expression for an approximation to the Fisher quantity index:

$$Q^F \approx \frac{[(\tilde{p}^0 + p^1) \cdot (q^1 - q^0)]}{[\tilde{p}^0 \cdot q^0(1 + P^F)]} + 1 \quad \text{(A12)}$$

$$= \frac{2V_B}{[\tilde{p}^0 \cdot q^0(1 + P^F)]} + 1 \quad \blacksquare$$